

CALCULUS 30
PROTOTYPE FINAL EXAM
JUNE 25, 2010

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Prototype

Name: _____

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TOTAL: 138 MARKS

This exam is closed book with one exception: Students are allowed one 8½" x 11" page of notes. Graphing calculators are allowed. Time allowed: 2 hours, 30 minutes. Please circle your final answers to the problems.

1. Evaluate the following limits.

$$\lim_{x \rightarrow 3} \frac{x^2 - 16}{x - 4}$$

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 4}{x - 16}$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 7x}{x - 1}$$

2. State the definition of the derivative of y with respect to x .

3. Use the definition of the derivative to differentiate $y = 5x^3 + 4x$.

4. Find the first derivative, $\frac{dy}{dx}$, of the following function. Factor your final answer over the set of rational numbers.

a. $y = 6x^2 - 2x + 3$

b. $y = \frac{2}{3}x^4 + \frac{4}{7}x^3 + \frac{1}{2}x^2 + 3x + 6$

c. $y = (x^2 + 8x - 4)^2$

d. $y = (x + 8)\sqrt{x^2 + 4x}$

e. $y = \frac{5-3t^2}{3t^2-2t}$

f. $y = \frac{5x+6}{(x^2-2)^2}$

5. Find $f'(x)$ and $f''(x)$ for this function:

$$f(x) = 3x^5 + 2x^4 - 7x^3 - 2.5x^2 + 7x - 201$$

6. Use implicit differentiation to find the derivative, $\frac{dy}{dx}$, of these functions with respect to x .

$$y^2 = 7xy - 9$$

$$5xy^2 + y^3 - 2y^4 = 3x^2$$

7. Find the slope of the tangent line at the point $(0, 1)$ for the function:
 $y = (7x^2 + 3x - 4)^4$.

8. Find the critical number(s) for the function $f(x) = x^3 + 6x^2 + 9x - 4.34$.

9. Find the intervals in which the function is increasing or decreasing. Find the coordinates of any relative extrema. Use the first derivative test. You may wish to verify your results with a graphing calculator.

a. $f(x) = (x + 3)^2$

b. $y = x - \frac{2}{x}$

10. Determine the equations of all asymptotes of the function $y = \frac{x^2 + x}{x^3 - 27}$.

11. We can use derivatives to find equations to describe certain aspects of movement. For example, velocity can be described by the first derivative or y' .

a. What aspect does y'' represent? (2 marks)

12. For the function $f(x) = x^4 - 2x^3$, find the following:
- a sign analysis of $f'(x)$
 - the open intervals on which $f(x)$ is increasing and/or decreasing.
 - the critical numbers
 - the relative extrema
 - a sign analysis of $f''(x)$
 - the intervals on which $f(x)$ is concave up and concave down.
 - the coordinates of any inflection points.
 - the x and y intercepts.
 - the equations of any horizontal and vertical asymptotes.
 - a careful sketch of the function that supports all of the above features.

13. If gasoline costs \$1.25/L and you drive 25000 km per year, then your annual cost for gasoline will be given by the function $C(x) = 31250/x$ where x is the number of km/L that your car can travel.
- Find your annual gasoline costs if your car gets 4 km/L.
 - Find your annual gasoline costs if your car gets 10 km/L.
 - What is the average rate of change in your annual gasoline cost as x changes from 4 to 10?
 - What is the instantaneous rate of change in your annual fuel costs when $x = 5$?
14. A rectangular box-shaped garbage can with a square base and an open top is to be constructed using exactly 900 cm^2 of material. Find the dimensions of the box that will provide the greatest possible volume.

15. A cylindrical tank has a radius of 4m and a depth of 8m. It is being filled at the rate of $4 \text{ m}^3/\text{min}$. How fast is the surface rising?

- END OF EXAM -

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